EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Prove that the locus of the middle point of the chord of contact of tangents from any point of the circle $x^2 + y^2 = r^2$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by the equation $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2 = \frac{(x^2+y^2)}{r^2}$.

Sol.

2. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2). Find the area of the triangle that these tangents from with their chord of contact.

Sol.

3. A line through the origin meets the circle $x^2+y^2= a^2$ at P & the hyperbola $x^2 - y^2 = a^2$ at Q. Prove that the locus of the point of intersection of the tangent at P to the circle and the tangent at Q to the hyperbola is curve $a^4(x^2 - a^2) + 4x^2y^4 = 0$.

Sol.

4. The normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ drawn at an extremity of its latus rectum is parallel to an asymptote. Show that the eccentricity is equal to the square root of $(1+\sqrt{5})/2$.

Sol.

5. Ascertain the co-ordinates of the two points Q & R, where the tangents to the hyperbola $\frac{x^2}{45} - \frac{y^2}{20} = 1$ at the point P(9, 4) intersects the two asymptotes. Finally prove that P is the middle point of QR. Also compute the area of the triangle CQR where C is the centre of the hyperbola.

Sol.

6. A point P divides the focal length of the hyperbola $9x^2 - 16y^2 = 144$ in the ratio $S'P : PS = 2 : 3$ where S & S′ are the foci of the hyperbola. Through P a straight line is drawn at an angle of 135º to the axis OX. Find the points of intersection of this line with the asymptotes of the hyperbola.

Sol.

7. Find the length of the diameter of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ perpendicular to the asymptote of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ passing through the first & third quadrants.

Sol.

8. The tangent at P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of the asymptote in Q. Show that the locus of the mid point of PQ is a similar hyperbola.

Sol.

9. A transversal cuts the same branch of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in P,P′ and the asymptotes in Q, Q′.

Prove that (i) PQ = P′Q′ & (ii) PQ′ = P′Q

Sol.

10. From any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, tangents are drawn to another hyperbola which has the same asymptotes. Show that the chord of contact cuts off a constant area from the asymptotes.

Sol.

11. Through any point P of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a line QPR is drawn with a fixed gradient m, meeting the asymptotes in Q & R. Show that the product, $(QP) \cdot (PR) = \frac{a^2b^2(1+m^2)}{b^2 - a^2m^2}$.

Sol.

12. If a rectangular hyperbola have the equation, $xy = c^2$, prove that the locus of the middle points of the chords of constant length 2d is
$(x^2 + y^2)(x y - c^2) = d^2xy$.

Sol.

13. Prove that infinite number of triangles can be inscribed in the rectangular hyperbola, $x y = c^2$ whose sides touch the parabola, $y^2 = 4ax$.

Sol.

14. The normals at three points P, Q, R on a rectangular hyperbola $xy = c^2$ intersect at a point on the curve. Prove that the centre of the hyperbola is the centroid of the triangle PQR.

Sol.

15. Tangents are drawn from any point on the rectangular hyperbola $x^2 - y^2 = a^2 - b^2$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that these tangents are equally inclined to the asymptotes of the hyperbola.

Sol.